nag_real_qr (f01qcc)

1. Purpose

nag_real_qr (f01qcc) finds the QR factorization of the real m by n matrix A, where $m \geq n$.

2. Specification

```
#include <nag.h>
#include <nagf01.h>

void nag_real_qr(Integer m, Integer n, double a[], Integer tda,
     double zeta[], NagError *fail)
```

3. Description

The m by n matrix A is factorized as

$$
\begin{aligned}
A &= Q\begin{pmatrix} R \\ 0 \end{pmatrix} \quad \text{when } m > n, \\
A &= QR \qquad\quad \text{when } m = n,
\end{aligned}
$$

where Q is an m by m orthogonal matrix and R is an n by n upper triangular matrix. The factorization is obtained by Householder's method. The kth transformation matrix, Q_k, which is used to introduce zeros into the kth column of A is given in the form

$$Q_k = \begin{pmatrix} I & 0 \\ 0 & T_k \end{pmatrix}$$

where

$$
\begin{aligned}
T_k &= I - u_k u_k^T, \\
u_k &= \begin{pmatrix} \zeta_k \\ z_k \end{pmatrix},
\end{aligned}
$$

ζ_k is a scalar and z_k is an $(m-k)$ element vector. ζ_k and z_k are chosen to annihilate the elements below the triangular part of A.

The vector u_k is returned in the $(k-1)$th element of the array **zeta** and in the $(k-1)$th column of **a**, such that ζ_k is in **zeta**$[k-1]$ and the elements of z_k are in **a**$[k][k-1], \ldots,$**a**$[m-1][k-1]$. The elements of R are returned in the upper triangular part of **a**. Q is given by

$$Q = (Q_n Q_{n-1} \cdots Q_1)^T.$$

Good background descriptions to the QR factorization are given in Dongarra *et al*(1979) and Golub and Van Loan (1989).

4. Parameters

m

Input: m, the number of rows of A.
Constraint: $\mathbf{m} \geq \mathbf{n}$.

n

Input: n, the number of columns of A.
When $\mathbf{n} = 0$ then an immediate return is effected.
Constraint: $\mathbf{n} \geq 0$.

a[m][tda]

Input: the leading m by n part of the array **a** must contain the matrix to be factorized.
Output: the n by n upper triangular part of **a** will contain the upper triangular matrix R and the m by n strictly lower triangular part of **a** will contain details of the factorization as described in Section 3.

tda

Input: the second dimension of the array **a** as declared in the function from which nag_real_qr is called.
Constraint: **tda** $\geq$ **n**.

zeta[n]

Output: **zeta** $[k-1]$ contains the scalar ζ_k for the kth transformation. If $T_k = I$ then **zeta**$(k-1) = 0.0$, otherwise **zeta** $[k-1]$ contains ζ_k as described in Section 3 and ζ_k is always in the range $(1.0, \sqrt{2.0})$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_2_INT_ARG_LT

On entry, **m** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These parameters must satisfy **m** $\geq$ **n**.
On entry, **tda** = ⟨*value*⟩ while **n** = ⟨*value*⟩. These parameters must satisfy **tda** $\geq$ **n**.

NE_INT_ARG_LT

On entry, **n** must not be less than 0: **n** = ⟨*value*⟩.

6. Further Comments

The approximate number of floating-point operations is given by $2n^2(3m-n)/3$.

6.1. Accuracy

The computed factors Q and R satisfy the relation

$$Q\begin{pmatrix} R \\ 0 \end{pmatrix} = A + E$$

where $\|E\| \leq c\epsilon\|A\|$, and ϵ is the ***machine precision***, c is a modest function of m and n and $\|.\|$ denotes the spectral (two) norm.

6.2. References

Dongarra J J, Moler C B, Bunch J R and Stewart G W (1979) *LINPACK Users' Guide* SIAM, Philadelphia.
Golub G H and Van Loan C F (1989) *Matrix Computations* (2nd Edn) Johns Hopkins University Press, Baltimore.
Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Clarendon Press, Oxford.

7. See Also

nag_real_apply_q (f01qdc)
nag_real_form_q (f01qec)

8. Example

To obtain the QR factorization of the 5 by 3 matrix

$$A = \begin{pmatrix} 2.0 & 2.5 & 2.5 \\ 2.0 & 2.5 & 2.5 \\ 1.6 & -0.4 & 2.8 \\ 2.0 & -0.5 & 0.5 \\ 1.2 & -0.3 & -2.9 \end{pmatrix}.$$

8.1. Program Text

```
/* nag_real_qr(f01qcc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagf01.h>

#define MMAX 20
#define NMAX 10

main()
{
  Integer tda = NMAX;

  double zeta[NMAX], a[MMAX][NMAX];
  Integer i, j, m, n;

  Vprintf("f01qcc Example Program Results\n");
  Vscanf(" %*[^\n]"); /* skip headings in data file */
  Vscanf(" %*[^\n]");

  Vscanf("%ld%ld", &m, &n);
  if (m > MMAX || n > NMAX)
    {
      Vprintf("m or n is out of range.\n");
      Vprintf("m = %2ld, n = %2ld\n", m, n);
    }
  else
    {
      Vscanf(" %*[^\n]"); /* skip next heading */
      for (i = 0; i < m; ++i)   /* Read matrix A */
        for (j = 0; j < n; ++j)
          Vscanf("%lf", &a[i][j]);

      /* Find the QR factorization of A */
      f01qcc(m, n, (double *)a, tda, zeta, NAGERR_DEFAULT);

      Vprintf("QR factorization of A\n\n");
      Vprintf("Vector zeta\n");
      for (i = 0; i < n; ++i)
        Vprintf(" %8.4f", zeta[i]);
      Vprintf("\n\n");
      Vprintf("Matrix A after factorization (upper triangular part is R)\n");
      for (i = 0; i < m; ++i)
        {
          for (j = 0; j < n; ++j)
            Vprintf(" %8.4f", a[i][j]);
          Vprintf("\n");
        }
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
f01qcc Example Program Data
Values of m and n.
  5     3
Matrix A
  2.0   2.5   2.5
  2.0   2.5   2.5
  1.6  -0.4   2.8
  2.0  -0.5   0.5
  1.2  -0.3  -2.9
```

8.3. Program Results

```
f01qcc Example Program Results
QR factorization of A

Vector zeta
   1.2247   1.1547   1.2649

Matrix A after factorization (upper triangular part is R)
  -4.0000  -2.0000  -3.0000
   0.4082  -3.0000  -2.0000
   0.3266  -0.4619  -4.0000
   0.4082  -0.5774   0.0000
   0.2449  -0.3464  -0.6325
```